Jiro Kodaira
Minami-Tateya
&
France-Japan Collaboration
(The need for NLO)

Fawzi Boudjema
LAPTH, Annecy
New Technique for Calculating Amplitudes

Calculational Point of View
1. Loop Calculation (Precision)
2. Multi - Log Amplitude (Background)
3. Gluons

Cascade Decay to Light (01) Particles

Backgrounds (and many signals)

Tight relation

Understanding of providing

Activations for

Many ultra-relativistic (massless)

Particles

- Especially quarks and gluons at

QCD

J. Kodaira (Riken) at Annecy

Annecy, June 2006
France Japan Collaboration Meeting
New Technique for Calculating Amplitudes

J. Kodaira (KEK) at Annecy

Calculational Point of View

1. Loop Calculation (Precision)
2. Multi – Leg Amplitude (Background)
   ex. Gluino

Cascade Decay to Light (SM) Particles

Backgrounds (and many signals) require detailed understanding of scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD
discovering supersymmetry

• the dominant production of superparticles at LHC is through pairs of gluinos and squarks

• their cascade decays produce high energy jets and large “missing energy” from neutralinos

• a simple discriminant for supersymmetry searches is the effective mass defined as

\[ M_{\text{eff}} = E_{T}^{\text{miss}} + \sum_{i=1}^{4} P_{T}^{\text{jet}} \]

• an excess of events with large \( M_{\text{eff}} \) could be the initial discovery of supersymmetry
- this strategy is backed up by this famous plot from the ATLAS TDR
- for 8 years, was used to make the case that LHC can discover supersymmetry after “a few weeks of running”
• at LHC, supersymmetry channels have large SM backgrounds from top, $Z$+jets, and $W$+jets

• showering Monte Carlos like Isajet and Pythia underestimate these backgrounds by up to a factor of ten in the signal region

• this was forgotten until recently, when better QCD theory tools became available
Result (1)  No lepton mode

Asai, ATLAS Japan

\[ m(\tilde{g}, \tilde{q}) \sim 1 \text{TeV} \]

Effective Mass(GeV) = m_{E_T} + \sum P_T(4jets)

1. Background increases by factor 2–4 than the PS prediction depending on M_{eff}.
2. Slope becomes more gentle, and similar to the signal ← important Point
3. All BG contributions are the same order. Not easy to understand BG using real data.
   → see Next Page.

Conclusion

[1] Backgrounds for SUSY search are estimated with the Matrix Element information.
   (1) The background increases by factor 2–4
   (2) Shape of Background becomes similar to signal.
      → BG estimation using real data is essential. We need to understand slope of distributions.
   (3) W/Z+Njets studies @ Tevatron give good information for us.
   (4) QCD multijet contributions are the same order of the other background processes: need more careful treatment.
      → Missing E_T is vital business.
It is not unlikely that NP signals will emerge from counting expt. which require control over SM and Background simulations in PS/ME mergers/matches. Total rates are still computed @LO not good enough!!, large scale dependence,..

overall K factors still not sufficient, may be distribution/cut dependent

Comparison to NLO:
validation of matching and testing of ME & PS (CKKW, MLM/Alpgen, Grace-NLO see YK)

NLO simulation needed
SM particles are produced in association with jets

- At LHC additional jets will be harder
  - Application of N(N)LO corrections in MC's, crucial for proper understanding of backgrounds
  - Not just a question of normalization!

However, the vast majority of physics studies are performed with LO ME + PS

- This may be ok for a discovery with narrow resonances or large excess of events
  - However, analysis restricted to simple cut on invariant mass distributions. Discriminating power is diminished
- But this fails for more complex searches involving jets
  - Present level of analyses not acceptable
  - Higher order corrections are crucial and need to be somehow implemented in our MC's

We are very happy to see NLO MC integrators like MCFM and NLO event generators like GRACE and MC@NLO adding new processes

- How many processes will be described to NLO by turn on?
- What will be the impact of NNLO corrections at the LHC?
1 (Many-)Particle production at NLO

A lot of processes with \( n \geq 3 \) particles in final states only known at LO

\[ \Rightarrow \] enormous amount of homework for theorists

State-of-the-art for NLO in theory:

- techniques for \( 2 \rightarrow 3 \) processes established;
  results known for several processes at hadron colliders:
  \[ pp \rightarrow 3 \text{jets}, V + 2 \text{jets}, V b \bar{b}, \gamma \gamma \text{jet}, t\bar{t}H, b\bar{b}H \]
  \[ \Rightarrow \] calculations still demanding

- \( 2 \rightarrow 4 \) processes are technical frontier;
  only two results for EW corrections in \( e^+e^- \) physics:
  \[ e^+e^- \rightarrow \nu\bar{\nu}HH, \quad e^+e^- \rightarrow 4f \]
  \[ \text{GRACE-1-loop (Boujema et al.) ’04} \quad \text{Denner et al. ’05} \]
  + some partial or toy-model results
  \[ \text{Bern et al., Binoth et al.} \]
  \[ \Rightarrow \] calculations very challenging + lengthy!

\[ \Rightarrow \] Theorists need a clear list of important processes
  including arguments for “why calculating what !?”

6 gluons amplitude

Eliis-Giele-Zanderighi 2006
Ingredients for NLO for M external legs

- **Most difficult:** One-loop RC to the M-leg partonic subprocess (Grace-loop)

- Tree-level to M+1-leg partonic subprocess (GRC4F for LEP2 big success, GRAPPA for LHC see YK)

- Efficient method to merge real and virtual, cancel IR (OK, see YK)

- Parton shower at NLO (merging)

Many of these steps handled by France-Japan Coll. and were being improved and optimized with Kodaira-san (QCD),

*Need for automatisation,*...
**Jets in photon-photon collisions: from TRISTAN to J/N-LC.**


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**Lepton pair productions with double hard photon emission**


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**GRC4F**

_Grc4f v1.1: A Four fermion event generator for e+ e- collisions._


Full 1-loop RC available for 2->3 and 2->4

\[ e^+e^- \rightarrow \nu\bar{\nu}H \quad (1350) \]
\[ e^+e^- \rightarrow t\bar{t}H \quad (2327) \]
\[ e^+e^- \rightarrow ZHH \quad (5417) \]
\[ e^+e^- \rightarrow e^+e^-H \quad (4470) \]
\[ e^+e^- \rightarrow \nu\bar{\nu}\gamma \quad v = \nu_\mu, \nu_\tau \]
\[ e^+e^- \rightarrow \nu\bar{\nu}HH \quad (19638) \]
\[ e^+e^- \rightarrow \nu_\tau^+\mu^-\bar{\nu}_\mu, \ u\bar{d}\bar{s} \]

Denner et al., NPB660(2003)289
Denner et al., PLB575(2003)290
You et al., PLB571(2003)163
Zhang et al., PLB578(2004)349

Also available on
www.sciencedirect.com

algebraic reduction

\[ \sum_{i=1}^{6} b_i \]

integrals with less legs
from reduction of tensor rank and
number of legs at the same time

non-trivial tensor structure
scalar 6-point function

reduction until set of basis integrals is reached
basis integrals: 2-, 3- and 4-point functions ⇒ known

Golem

our approach:
semit-numeralical method implemented in program GOLEM
(General One-Loop Evaluator for Matrix elements)
[Binoth, Guiffanti, Guillet, GH, Karg, Kauer, Pilon, Reiter, …]

main features:

- use algebraic reduction until singularities can be easily isolated (dim. reg.)
- inverse Gram determinants can be completely avoided by choosing a convenient set of (non-scalar) basis integrals (*stop reduction before it generates problems*)
- valid for massless and massive particles and for (in principle) arbitrary number of legs
<table>
<thead>
<tr>
<th>process ( (V \in {Z, W, \gamma}) )</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( pp \rightarrow V V \text{ jet} )</td>
<td>( t\bar{t}H ), new physics</td>
</tr>
<tr>
<td>2. ( pp \rightarrow t\bar{t} b\bar{b} )</td>
<td>( t\bar{t}H )</td>
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<tr>
<td>3. ( pp \rightarrow t\bar{t} + 2 \text{ jets} )</td>
<td>( t\bar{t}H )</td>
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<tr>
<td>4. ( pp \rightarrow V V b\bar{b} )</td>
<td>( VBF \rightarrow H \rightarrow VV ), ( t\bar{t}H ), new physics</td>
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<tr>
<td>5. ( pp \rightarrow V V + 2 \text{ jets} )</td>
<td>( VBF \rightarrow H \rightarrow VV )</td>
</tr>
<tr>
<td>6. ( pp \rightarrow V + 3 \text{ jets} )</td>
<td>various new physics signatures</td>
</tr>
<tr>
<td>7. ( pp \rightarrow V V V )</td>
<td>SUSY trilepton</td>
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</tbody>
</table>
We want NLO Monte Carlos!
Non-linear gauge fixing terms

\[ L_{GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi} (F^A)^2 \]

\[ F^\pm = \left( \partial^\mu + i e \tilde{\alpha} A^\mu + i \frac{e c_W}{s_W} \beta Z^\mu \right) W^\pm_\mu \]

\[ F^A = \partial^\mu A^\mu \]

\[ F^Z = \partial^\mu Z^\mu + \xi_Z \left( M_W \chi^\pm + \frac{e}{2s_W} \tilde{\delta} H \chi^\pm \pm i \frac{e}{2s_W} \tilde{\kappa} \chi_3 \chi^\pm \right) \]

\[ \text{Boudjema and Chopin Z.Phys. C73(1996) 85} \]
Non-linear gauge

- Numerator structure is the same as Feynman gauge
  → Loop integral library
- Vertices modified
- general values → #diagrams

$g^{\mu\nu}$ (for $\xi = 1$)

"old" usage → reduce #diagram
$\tilde{\alpha} = 1 \Rightarrow$ no $\mathcal{W}\chi$

Check gauge invariance → Independence on gauge parameters
Higgs@LC: tree cross sections

Main channels

WWH vs ZZH

Yukawa coupl.

Higgs Potential

$\sigma(\text{fb})$

$\sqrt{s}(\text{GeV})$

$M_H = 120 \text{GeV}$

$2 \rightarrow 3, 4$ processes important
Linear Gauge vs Non-Linear Gauge for $e^+e^- \rightarrow \nu\bar{\nu}HH$

<table>
<thead>
<tr>
<th>NLG Cuv=0</th>
<th>Cuv=100</th>
<th>LG Cuv=0</th>
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<tbody>
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Cascade Decay to Light (SM) Particles

Backgrounds (and many signals) require detailed understanding of scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD
2 → 3: only a few NLO results for hadronic collisions

\[ pp \rightarrow 3 \text{jets} \]
\[ pp \rightarrow V + 2 \text{jets} \]
\[ pp \rightarrow \gamma\gamma \text{jet} \]
\[ pp \rightarrow ttH, b\bar{b}H \]
\[ pp \rightarrow tt \text{jet} \]
\[ p\bar{p} \rightarrow W b\bar{b} \]

- Understand complex factorization at one loop and beyond + connection to Lagrangian?
- Higher loops?
- Massive partons (external fermions, scalars, …)
- Automatization
- Attack the wishlists...
**Full 1-loop RC available for 2->3 and 2->4**

- $e^+e^- \rightarrow \nu\bar{\nu}H$  
  (1350)  
  GRACE, PLB559(2003)252  
  Denner et al., NPB660(2003)289

- $e^+e^- \rightarrow ttH$  
  (2327)  
  GRACE, PLB571(2003)163  
  You et al., PLB571(2003)85  
  Denner et al., PLB575(2003)290

- $e^+e^- \rightarrow ZHH$  
  (5417)  
  Zhang et al., PLB578(2004)349

- $e^+e^- \rightarrow e^+e^-H$  
  (4470)  
  GRACE, PLB600(2004)65

- $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  
  $\nu = \nu_\mu, \nu_e$  
  GRACE, NIM A534(2004)334

- $e^+e^- \rightarrow \nu\bar{\nu}HH$  
  (19638)  
  GRACE, hep-ph/0510184

- $e^+e^- \rightarrow \nu_\tau\tau^+\mu^-\bar{\nu}_\mu, ud\bar{s}c$  
  Denner et al., hep-ph/0502063
Non-linear gauge fixing terms

\[ \mathcal{L}_{GF} \equiv - \frac{1}{\xi_W} F^+ F^- - \frac{1}{2 \xi_Z} (F^Z)^2 - \frac{1}{2 \xi_A} (F^A)^2 \]

\[ F^\pm \equiv (\partial_\mu \mp i e \alpha A_\mu \mp ig c_W \beta Z_\mu) W^{\mu+} \]
\[ + \xi_W \frac{g}{2} (\nu + \tilde{\delta} H \mp i \kappa \chi_3) \chi^+ \]

\[ F^Z \equiv (\partial \cdot Z + \xi_Z \frac{g}{2 c_W} (\nu + \bar{\epsilon} H) \chi_3)^2 \]

\[ F^A \equiv \partial \cdot A \]

6 gluons amplitude


Eliis-Giele-Zanderighi 2006